



DYNAMIC ANALYSIS OF ROTOR SYSTEM WITH ACTIVE MAGNETIC BEARINGS USING FINITE ELEMENT METHOD

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Abstract—The rotor is a key element of rotating machinery that can be used in different of mechanical devices such as gas turbines, industrial compressors, motors, and aero engines. The dynamic analysis of a rotor system with active magnetic bearings is presented in this paper. This system consists of a flexible shaft with a rigid disk and flexible bearings. An sys Workbench is used to study a 3D simulation of a finite element model. Correct estimation of the rotor's natural frequencies and mode shapes is required for rotating machinery safety procedures. In order to compute the natural frequencies and mode shapes of this system, in the first case it is assumed a free-free system in which no bearings are used, and the natural frequencies and mode shapes are calculated without rigid body modes. In the second case, the natural frequencies and mode shapes are determined with bearing stiffness. The natural frequencies are also evaluated using finite elements based on matlab code. A comparison between the two methods was carried out.

Keywords—(Finite element method; Ansys Workbench; Dynamic analysis; Active Magnetic Bearing).

I. INTRODUCTION

Rotating machines are nowadays the most important components in mechanical systems. Rotating components in these systems cause vibration. A thorough understanding of system behavior is required to reduce vibrations caused by rotating machinery. Magnetic bearings have a number of advantages in the performance of the rotating machine, including the ability to achieve extremely high angular velocity with no lubrication, high power density with no physical interaction between the bearings' rotors and stators, and operating without mechanical wear, among other important advantages. Because of these and other advantages, AMBs are frequently employed as a replacement for traditional mechanical bearings in many rotating equipment applications. As a result, several studies on rotor systems with active magnetic fields have been conducted.

The interaction of the rotor with active magnetic bearing can result in a variety of nonlinear rotor responses. The most

prevalent source of nonlinearity is the relationship between the active magnetic bearings and the coil current, electromagnetic forces, and the air gap between the rotor and the stator. Active magnetic bearings provide a magnetic force that is proportional to current squared and inversely proportional to air gap. There have been several articles in recent years on the dynamics study of the rotor with active magnetic bearings. The dynamics of a flexible rotor with active magnetic bearings have been investigated in this research. Finite element method was utilized for dynamic simulation of a flexible rotor with active magnetic bearings using Ansys Workbench [1].

Natural frequencies and mode shapes were used for predicting dynamic behaviors of rotor model with and without bearing conditions [2]. The finite element technique was used to study the dynamic analysis of the rotor system [3]. The dynamic analysis of the rotor shaft assembly for magnetically levitated motors has been studied. The dynamics of flexible rotor supported by AMB is analyzed by FEM [4]. A co-simulation model for high-speed rotating elastic rotors supported by active magnetic bearings has been presented using ADAMS and Matlab/Simulink. In the Ansys program, the rotor is simulated as a flexible element [5]. It was presented a complete design and modeling framework for a flexible high-speed rotor supported by active magnetic bearings. The essential structural elements were demonstrated in detail, containing the main dimensions and properties of the AMB elements and rotor. The rotor dynamics model was solved using the finite element method that is based on Timoshenko beam theory. Obtaining rotor dynamic properties via synthesis of natural frequency, and mode shapes [6]. A new laboratory test rig of the vertical axis flexible shaft supported by two active magnetic bearings is presented [7]. The 3D finite element analysis (FEA) is utilized to establish the natural frequencies and vibration modes of the magnetic bearing-rotor system of high-speed machine and to define bearing stiffness of 3D-FEA model by exciting experiment [8]. Using the finite element technique, the natural frequencies of a thick-disk rotor with increased stiffness are determined. Ansys Workbench is used to perform the thick disc modal analysis [9]. By using Ansys parametric design tool, natural frequency, and the amplitudes of rotor system have been determined [10]. To

evaluate the dynamic behavior of a simple horizontal rotor system, a finite element model is constructed. The rotor system contains of a single shaft that is supported at both ends by bearings, and two discs that are installed at different positions. The natural frequencies and mode shapes are computed by using a MATLAB code to solve the equations of the finite element rotary system [11]. The finite element method is used to construct a model of a rotor system with two discs. The gyroscopic effects are included in the system model, which is using to obtain the mode shapes utilizing Ansys Workbench [12]. A comparison of several methods for rotor AMB system vibration analysis has been performed [13]. In this study the finite element analysis FEA technique for rotor bearing foundation system analysis is used [14]. Steady-state and transient response of Jeffcott rotor with elastic bearings is produced [15]. A flexible rotor bearing system utilizing finite element technique is performed [16].

The dynamic analysis of a rotor system with active magnetic bearings is presented in this paper. Ansys Workbench is used to assess a model's 3D simulation. The natural frequencies are also evaluated using a finite element based on matlab code. A comparison between the two methods is performed.

II. EQUATION OF MOTION BASED ON THE FINITE ELEMENT METHOD

The finite element method (FEM) can be used to model complex rotor systems by dividing them into small finite-dimension elements [2]. The rotor system mathematical model consists of a flexible shaft with a single rigid disc and flexible bearings. The gyroscopic moments and rotator inertia are taking into account. The system is represented schematically in Fig.1.

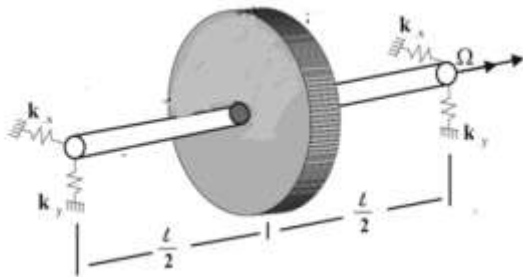


Fig.1. Flexible shaft with rigid disk and flexible bearing

A. Equation of Motion of the Rigid Disk

In Fig.2 the displacement vector of disk is given by: [9]

$$u_{1d} = \{x, \theta_y\}^T \quad (1)$$

$$u_{2d} = \{y, \theta_x\}^T \quad (2)$$

And the equation of motion of the rigid disk is expressed as following:

$$[M_d]\{\ddot{u}_{1d}\} + \Omega [J_d] \{\dot{u}_{1d}\} = \{Q_{1d}\} \quad (3)$$

$$[M_d]\{\ddot{u}_{2d}\} - \Omega [J_d] \{\dot{u}_{2d}\} = \{Q_{2d}\} \quad (4)$$

$$[M_d] = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & I_d & 0 \\ 0 & 0 & 0 & I_d \end{bmatrix} \quad (5)$$

Where Ω is the rotational speed of the disk, Q_d is the generalized forces vector, and M_d and J_d are the mass and gyroscopic matrices, respectively.

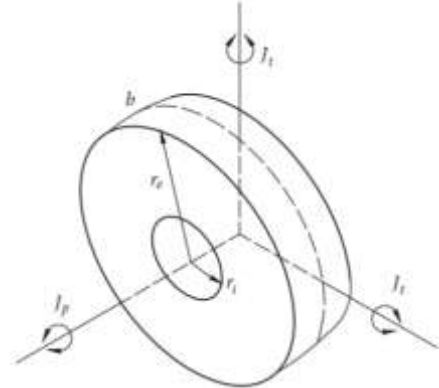


Fig. 2. Moment of inertia of the disk

B. Finite Element of the Shaft

In Fig.3 The finite element of the flexible shaft which supported by a flexible bearing contains eight degrees of freedom which is characterized by two nodes. The following are its parameters: (S, l) is the cross section area of the shaft element, and the element length of the shaft, respectively [9]. The displacement vector of the nodal of the shaft element is presented by:

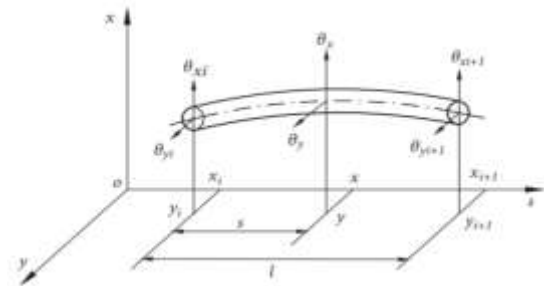


Fig.3 shaft finite element

$$u_{1s} = \{x_i, \theta_{y_i}, x_{i+1}, \theta_{y_{i+1}}\}^T \quad (6)$$

$$u_{2s} = \{y_i, \theta_{x_i}, y_{i+1}, \theta_{x_{i+1}}\}^T \quad (7)$$

The overall number of nodes in the rotor shaft increases to (N+1) when it is divided into (N) finite elements. And the equation of motion of the shaft finite element can be written as [17].

$$[M_S]\{\ddot{u}_{1s}\} + \Omega [J_s] \{\dot{u}_{2s}\} + k_s \{u_{1s}\} = \{Q_{1s}\} \quad (8)$$

$$[M_S]\{\ddot{u}_{2s}\} - \Omega [J_s] \{\dot{u}_{1s}\} + k_s \{u_{2s}\} = \{Q_{2s}\} \quad (9)$$



Where (M_s) is the mass matrix of shaft, (J_s) is the moment of inertia of shaft, and (k_s) is the stiffness matrix of shaft element that are expressed as:

$$[K_s] = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \quad (10)$$

$$[M_s] = \frac{\rho AL}{420} \begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L \\ 54 & 13L & 156 & -22L \\ -13L & -3L & -22L & 4L^2 \end{bmatrix} \quad (11)$$

B. Equation of Motion of Rotor System

The equation of motion of the rotor system with active magnetic bearings is expressed as following:

$$[M]\{\ddot{u}\} + [C] + \Omega[G]\{\dot{u}\} + [K]\{u\} = f_{AMB} + F_g \quad (12)$$

M and G denote the mass matrix and gyroscopic matrix, respectively, which include the shaft and the rigid disc. C is the damping matrix, while K is the shaft stiffness matrix. F_g is the gravity force vector, and f_{AMB} is the electromagnetic force of the active magnetic bearing.

The control current i_x and i_y is provided by the controller in the electromagnet. And the magnetic forces f_{AMB} which lift the rotor are produced by the electromagnet coils. The magnetic forces are shown as following [18, 19].

$$f_{AMB} = \begin{bmatrix} f_x \\ f_y \end{bmatrix} = \begin{bmatrix} k_{ix} & 0 \\ 0 & k_{iy} \end{bmatrix} \begin{bmatrix} i_x \\ i_y \end{bmatrix} - \begin{bmatrix} k_{sx} & 0 \\ 0 & k_{sy} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \\ = k_i \begin{bmatrix} i_x \\ i_y \end{bmatrix} - k_s \begin{bmatrix} x \\ y \end{bmatrix} \quad (13)$$

The current stiffness k_i and the position stiffness k_s matrices for both active magnetic bearings AMB_s in x and y directions is shown as following: [20]

$$[k_s] = \begin{bmatrix} K_{sx1} & 0 & 0 & 0 \\ 0 & K_{sy1} & 0 & 0 \\ 0 & 0 & K_{sx2} & 0 \\ 0 & 0 & 0 & K_{sy2} \end{bmatrix} \quad (14)$$

$$[k_i] = \begin{bmatrix} K_{ix1} & 0 & 0 & 0 \\ 0 & K_{iy1} & 0 & 0 \\ 0 & 0 & K_{ix2} & 0 \\ 0 & 0 & 0 & K_{iy2} \end{bmatrix} \quad (15)$$

Where, k_{ix} and k_{iy} represent the current stiffness, k_{sx} and k_{sy} express position stiffness, i_x and i_y denote control current, x and y are the displacement. The equations that used

to determined k in x and y directions are represented as following:

$$k_{ix} = \frac{\mu_0 N^2 A_g i_{xo}}{g_s^2} \quad (16)$$

$$k_{iy} = \frac{\mu_0 N^2 A_g i_{yo}}{g_s^2} \quad (17)$$

$$k_{sx} = \frac{\mu_0 N^2 A_g i_{xo}^2}{g_s^3} \quad (18)$$

$$k_{sy} = \frac{\mu_0 N^2 A_g i_{yo}^2}{g_s^3} \quad (19)$$

N is the number of turns of coil; A_g is the cross-section area of air gap, μ_0 is the magnetic permeability of air which equals to $4\pi \times 10^{-7}$ Vs/Am; g_s is the length of air gap.

D. Equation of Motion for Determining Natural Frequencies and Mode Shapes of Rotor

To determine natural frequencies and mode shapes of rotor, damping and the external force is neglected, and Equation 12 then becomes the free equation of motion of rotor system as shown below:

$$[M]\{\ddot{u}\} + [K]\{u\} = 0 \quad (20)$$

Where M is the global inertia matrix, K is the stiffness matrix, it should be noted that the inertia matrix M contains the inertia effect of both shaft and disk. K contains the stiffness of shaft and the stiffness of bearing.

$$M = M_s + M_d \quad (21)$$

$$k = k_s + k_b \quad (22)$$

The solution to Equation 20 is assumed to be as follows: $u = U \exp(j\omega t)$, and the following is a characteristic equation:

$$(k - \omega^2 M) U = 0 \quad (23)$$

Because not all amplitudes for a free vibration rotor are equal to zero, the only condition for a nonzero solution in Equation (23) is that the coefficient of the determinant is equal to zero.

$$(k - \omega^2 M) = 0 \quad (24)$$

The natural frequencies of the rotor are thus represented by the characteristic values of Equation 24, and the nonzero vectors of Equation 23 correspond to the appropriate vibration modes.

III. MODEL ANALYSIS OF ROTOR SYSTEM WITH AMB USING ANSYS WORKBENCH

Active magnetic bearings are becoming more popular in rotating industrial applications, as a substitute of the traditional mechanical bearings. Consequently, several researches have been done in rotor active magnetic bearing systems analysis. In this work the 3D finite element model of rotor active magnetic bearing systems is constructed in Ansys Workbench, which is presented in Fig.4. The finite element

mesh is automatically produced, and the analyzing model has 3372 nodes and 1596 elements as shown in Fig.5. Model analysis is utilized to identify the vibration properties of a structure under free vibration, such as natural frequencies and mode shapes. Table-2. Shows the dimensions of rotor active magnetic bearing system.

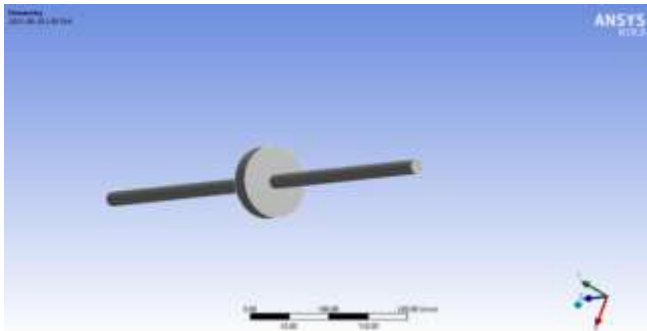


Fig.4. Structure of rotor AMB bearing system

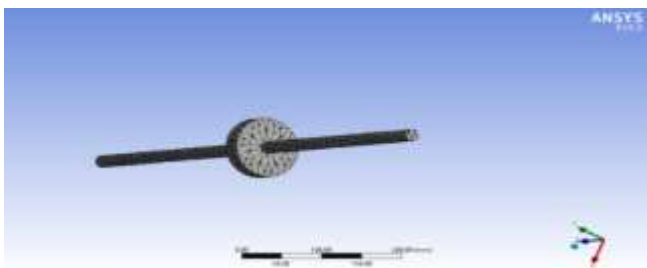


Fig.5. Finite element grid of rotor AMB bearing system

Parameter	Value
diameter of disc(mm)	100
width of disc(mm)	20
diameter of shaft(mm)	20
Length of shaft(mm)	620
Material	Structure steel
Density (kg/m ³)	7850
Modulus of elasticity(pa)	2×10^{11}

Table-1. Dimension of rotor bearing system

IV. SIMULATION RESULTS

A. Natural Frequencies and Mode Shapes of Rotor System in the Case of Free-Free System

Determining the natural frequencies and mode shapes is a necessary stage for predicting the dynamic behavior. The first step to calculate the natural frequencies and mode shapes of a free-free system (no bearings) and angular velocity $\Omega = 0$. The first six modes of free-free system are zero or closed to zero. The first two natural frequencies (with no rigid body modes) are 201.58 Hz, and 624.73 Hz. They are presented with mode shapes in Fig. (6 and 7).

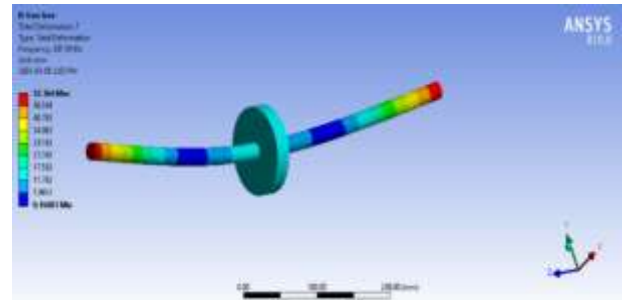


Fig.6. 1st mode shape of rotor bearing system at natural frequency=201.58 Hz

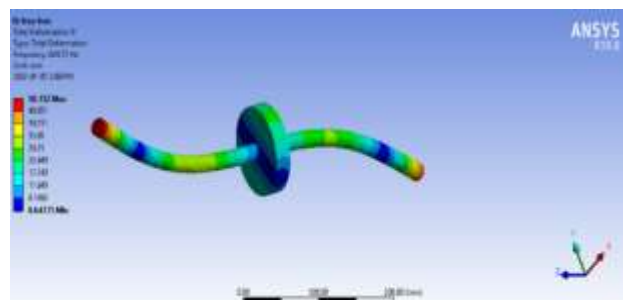


Fig.7. 2nd mode shape of rotor bearing system at natural frequency=624.73Hz

B. The Influence of Bearing Parameters in Stiffness of Bearing

The equation of electromagnetic force in AMB can be linearized as the following equation:

$$F(s,i) = k_i i - k_s s \quad (25)$$

Where F denotes the linear function of current and displacement, k_s and k_i describe the displacement and current stiffness of the magnetic bearings. The stiffness of the active magnetic bearing is calculated by bearing parameters which are shown in the following table (2).

Parameter	Value
number of turns of single pole	130
air gap length (mm)	0.5
coil current(A)	1
permeability of vacuum(N/A ²)	$4\pi \times 10^{-7}$
area of air gap(mm ²)	335.5×10^{-6}
current stiffness(N/A)	114
displacement stiffness(N/mm)	850

Table-2.Parameters of AMB

The stiffness of the active magnetic bearings is divided into current stiffness k_i and displacement stiffness k_s as shown in the equation (25). The value of the bearing stiffness depends



on several parameters, including coil current, air gap, and number of turns as explained in equation (16 to 19). Fig.8 described the effect of changing number of turns on current stiffness k_i and displacement stiffness k_s . It is noted that current stiffness and displacement stiffness increases with the increase of number of turns. For current = 1 A, and air gap = 0.5 mm the current stiffness and displacement stiffness at number of turns = 100 are 16.86 N/A, 33.73 N/m, respectively and at number of turns = 1000 are 1.7×10^3 N/A, 3373 N/m, respectively.

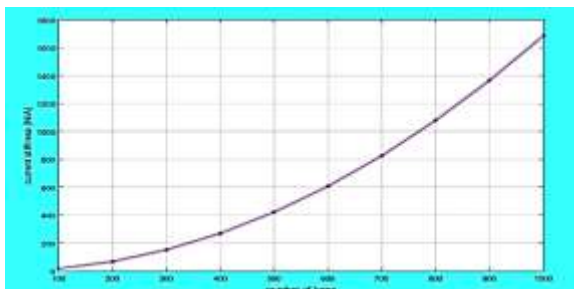
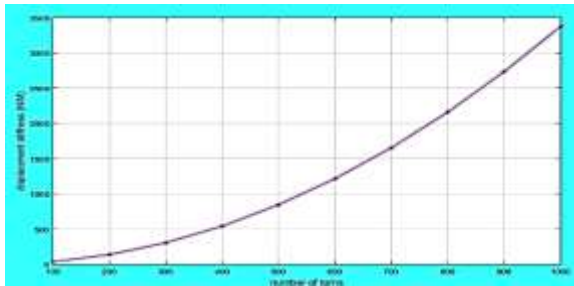


Fig.8. The displacement stiffness and current stiffness vs number of turns

Fig.9 shows the effect of air gap changing on the current stiffness k_i and displacement stiffness k_s . It has been observed that the current stiffness and displacement stiffness decrease with the increases of air gap. For current 1A and number of turns = 260 the current stiffness and displacement stiffness at air gap = 0.1 mm are 2.85×10^3 N/A, 2.85×10^4 N/mm, respectively and at air gap = 1.2 mm are 19.79 N/A, 16.49 N/mm, respectively.

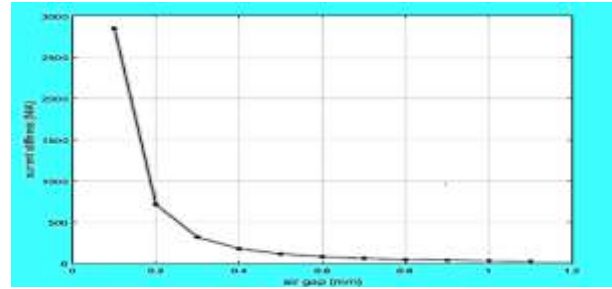
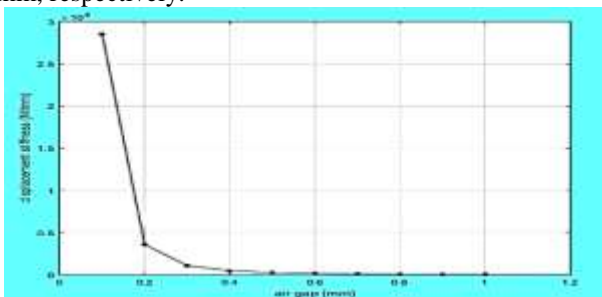


Fig.9. The displacement stiffness and current stiffness vs air gap

Fig.10 presents the effect of changing the current on the current stiffness k_i and displacement stiffness k_s . It has been found that the current stiffness k_i and displacement stiffness k_s increase with the increase of current. For air gap = 0.5 mm and number of turns = 260 the current stiffness, and displacement stiffness at $i = 0$ A are 0 N/A, 0 N/m, respectively and at $i = 6$ A are 684 N/A, 8208 N/m, respectively.

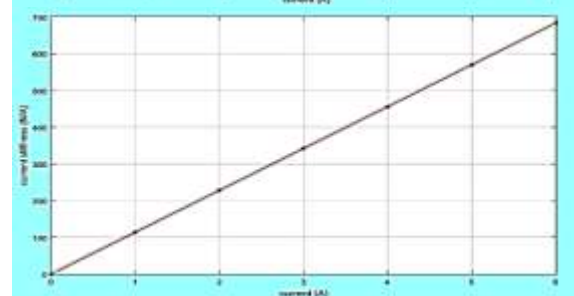
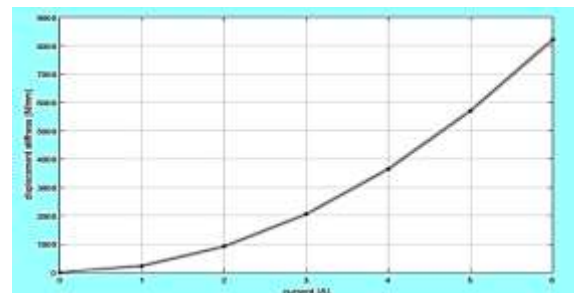


Fig.10. The displacement stiffness and current stiffness vs current

C. Natural Frequencies and Mode Shapes of Rotor System with Bearing Stiffness

In fact, in the magnetic bearings the rotor's boundary conditions really aren't free. The electromagnetic forces that support the shaft with active magnetic bearing are dependent on the control current in the coils of the stator and the air gap. The stiffness of bearing which dependent on the control components, can also be used to define this support condition. The stiffness of the elastically supported shaft can be computed for a certain rotor system operational point and then utilized to calculate natural frequencies and mode shapes. Fig.11 has been shown the rotor system with bearing stiffness.

The calculated bearing stiffness for the model was $k_s=850$ N/mm and $k_f=114$ N/A when the rotating speed was zero. The natural frequencies of the model with new modes became 63.05Hz, 260.74Hz, 403.21Hz, and 718.68Hz. They are represented with respective mode shapes in Fig. (12 to 15).

In this case, four natural frequencies were obtained due to the increase in the model modes number. The change in the stiffness of the bearing during the analysis is due to the stiffness coefficient of the control model, which changes significantly during the analysis. It is clear that with the increase in the stiffness of the bearing, the natural frequencies 201.58Hz, and 624.73Hz which is shown in Fig. (6 and 7) are increased. Compared to the original 0 frequency of the rigid mode shape which became 63.05 Hz and 260.74 Hz with its mode shapes presented in Fig. (12 to 15) the new mode shapes are now distinguished by a rigid motion which is a result of added bending. The natural frequencies were 260.74Hz (201.58Hz), 718.68Hz (624.73Hz). It can be shown that as bearing stiffness is increased, higher modes do not differ much. Whenever the bearing stiffness is increased, the natural frequencies also increased with the mode shapes changing. Eventually, the bearing stiffness is extremely high, and the mode shapes are also described by zero displacement at the bending position, which corresponds to the fixed boundary conditions.

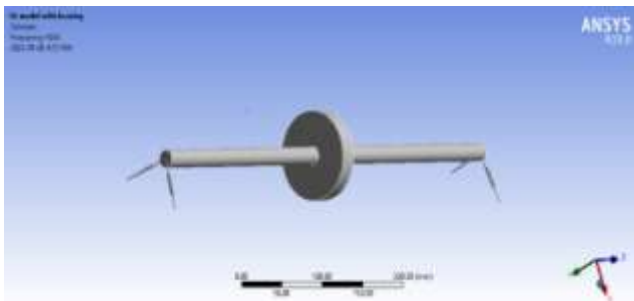


Fig.11. Structure of rotor system with bearing stiffness

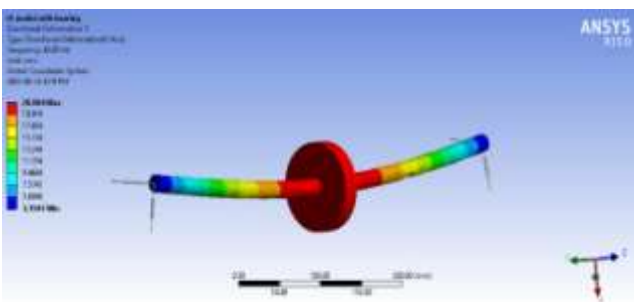


Fig.12. 1st mode shape of rotor system with bearing stiffness at natural frequency=63.05Hz

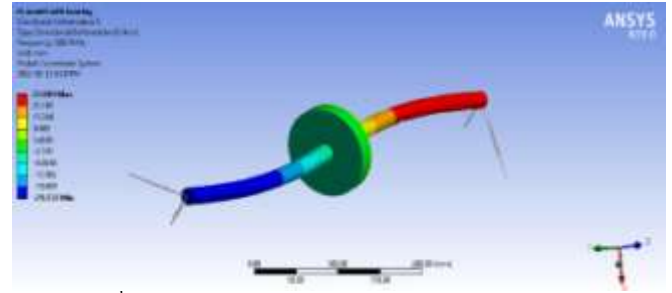


Fig.13. 2nd mode shape of rotor system with bearing stiffness at natural frequency = 260.74Hz

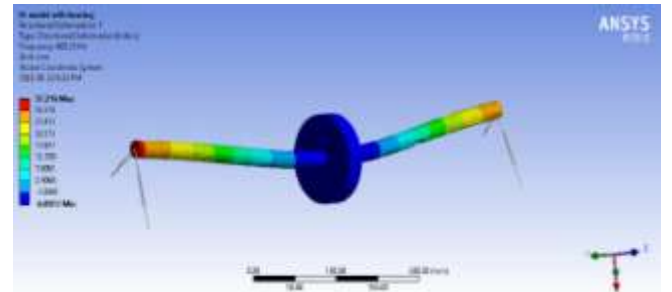


Fig.14. 3rd mode shape of rotor system with bearing stiffness at natural frequency = 403.21Hz

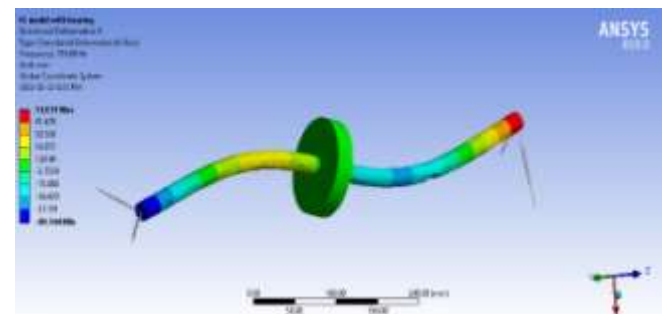
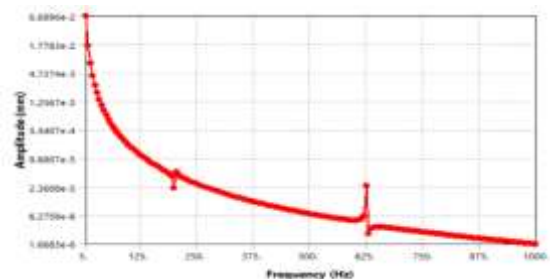


Fig.15. 4th mode shape of rotor system with bearing stiffness at natural frequency = 718.68Hz

D. Frequency Response of Rotor System

Fig.16 represent the frequency response of a flexible shaft at the free-free boundary condition in (x, y, z) direction. It can be noticed that the rotor response peak happened at 201.58 Hz and 624.73 Hz. At this time the rotor will undergo resonance phenomenon.



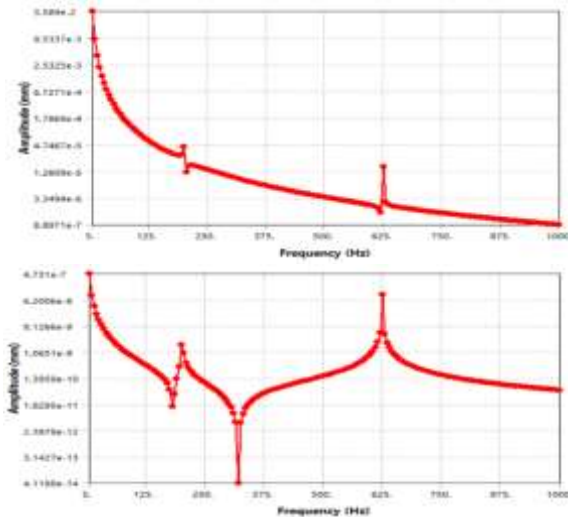


Fig.16. Frequency response of a rotor bearing system in (x, y and z) direction

V. COMPARISON BETWEEN THE RESULTS OBTAINED FROM ANSYS WORKBENCH AND FINITE ELEMENT BASED ON MATLAB CODE

The comparison between the first three natural frequencies of rotor system with bearing stiffness that obtained from Ansys Workbench and finite element based on Matlab code are presented in tables (3) and Fig.17. The percent errors of natural frequencies of Ansys Workbench by finite element based on Matlab code about 3% in the first natural frequency, 0.17% in the second natural frequency and 8.9% in the third natural frequency. Very close agreement has been obtained of the first three natural frequencies that obtained from the two methods.

Order	finite element calculation	Ansys Workbench results	Error
1	65HZ	63.05HZ	3%
2	260.3HZ	260.74HZ	0.17%
3	439.3HZ	403.21HZ	8.9%

Table-3. Natural frequencies of rotor by two different methods and errors

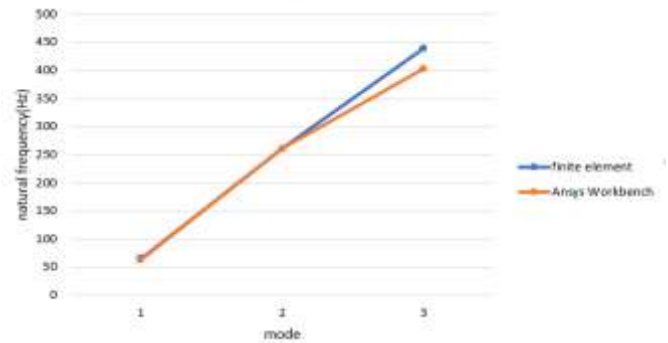


Fig.17. Comparison between results from finite element Matlab code and Ansys Workbench

VI. CONCLUSIONS

The dynamic analysis of rotor active magnetic bearings system is carried out in this paper. Natural frequencies and mode shapes are evaluated in the case of free-free system and with AMB stiffness at rotational speed $\Omega=0$. Natural frequencies increase with the increase of bearing stiffness. The higher modes do not differ much when the bearing stiffness increases. Stiffness of AMB increases with the increase in both current and number of turns and decreases with the increase in the air gap. Frequency response of free-free rotor system has been presented. Natural frequencies of rotor active magnetic bearings system are evaluated using Ansys Workbench and finite element based on Matlab code. Comparison between results that obtained from Ansys Workbench and finite element and the error percentages between them has been illustrated. Very close agreement has been obtained. The results which obtained from Ansys Workbench are more accurate.

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